A Comprehensive Insight into Lorentzian Geometry

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Preface

Lorentz space is a mathematical model developed specifically to explain event in the fields of electricity and magnetism. This model first emerged in the late 19th century, especially with the development of James Clerk Maxwell's theory of electromagnetism.

However, this theory was placed in a broader context with Albert Einstein's particular theory of relativity. Einstein suggested that electromagnetic fields can be perceived in different ways by different observers and that this perception may vary depending on the speed of moving objects. These ideas can be expressed in mathematical transformations known as Lorentz transformations. Mathematician Hermann Minkowski developed Minkowski space in the early 20th century. Based on Albert Einstein's theory of relativity, Minkowski suggested that time and space are interdependent, and this dependence can be expressed in a four-dimensional structure.

Lorentz space and Minkowski space have similar properties but different concepts and mathematical structures. Lorentz space is a concept used for the mathematical modeling of space-time in the special theory of relativity. Lorentz space represents four-dimensional space-time and is based on the special theory of relativity, which states that measurements of time depend on the movement of the observer.

Minkowski space is a four-dimensional mathematical model of space and time and is used in special relativity theory. It states that, like the Lorentz space, measurements of time and space depend on the movement of the observer.

That is, Lorentz space and Minkowski space are not the same space, but they use similar mathematical structures and concepts. Minkowski space is a special type of Lorentz space, and Lorentz space is a concept used in the special theory of relativity.

Metric concepts are defined according to the inner product after defining the inner product in Lorentz space. Since the Lorentz metric is not positive definite like the Euclidean metric, this situation occurs the vector diversity. Vectors are classified as timelike, spacelike, and null vectors in Lorentz space. This diversity among vectors has led to various angles, triangles, and all concepts in the metric sense. For this reason, it is very difficult to ensure the integrity of the issue related to the Lorentz space.

This book, which is a guide about the Minkowski space, has been prepared with two main components in mind. While the first one of them is detecting and correcting many present signs, vectors, and angle errors from the most basic to the recent studies in Minkowski space, the second one is the existence of the original parts. Generally, the studies were not paid attention to the vector types and ignored the harmony of the inner and vector products. There are also similar issues with curve types. Most of the results were given without paying attention to the curve types. In this book, we stated the causes of this literature errors and then made the essential corrections. To avoid disturbing the integrity of the subject, the original parts are not given in separate sections. According to the flow, these parts are highlighted without reference in the relevant section.

This book consists of the topics of linear algebra, differential geometry, analytic geometry, theory of curves, theory of surfaces, kinematics and manifolds related to Lorentz-Minkowski space. Lorentz space is an essential field that should know for all researchers interested in mathematics and especially geometry. There are fundamental titles that researchers will encounter when they study on Lorentz space. Some of these are definitions of angles between vectors, triangle classifications, cross-product, and the concept of the angle between vectors. But these parts are unclear in the Lorentz field. This book has been prepared as a detailed research book that eliminates confusion and unclarity.

To summarize, the two most important features of the book for the audience are as follows:

1) In this work, which was written with the same notation from beginning to end, the confusion to be experienced during the literature review has been minimized.

2) The original parts of the book will form the basis of future work.

This book consists of 10 chapters. The first chapter gives information about the purpose of writing the book and its content. The second chapter provides the information we need to define the Lorentz metric under the title of scalar product space.

The third chapter is in which the Lorentz space is introduced in detail. Vector classification, properties provided by vectors are given. Which vectors can be orthogonal to each other under what conditions is examined as a separate topic in each of the spaces $\mathbb{R}^n_1$,$\mathbb{R}^3_1$, $\mathbb{R}^3_2$ and $\mathbb{R}^2_1$. Schwartz and triangle inequality is explained in an explanatory and systematic way for different types of triangles.

The most problematic angle and triangle concepts for Lorentz space are examined in detail in the fourth part of the study. This section explains the errors in the literature with their corrected reasons. This section examines the space $\mathbb{R}^3_1$ and Lorentz plane under separate headings. All possible types of triangles and all right triangles are given. These triangle hyperbolic sine and cosine theorems are also explained one by one. In addition, the pedoe inequality, another original content of the book, has been examined for all triangles. So, the angle is an important part that will end the triangle confusion in Lorentz space.

The content of the fifth chapter is the analytical geometry part. In this section, the equations of timelike and spacelike lines are given. Different from the line equation in the literature, equations were found, and their reasons were explained. In addition, the vertical projection of different types of vectors on each line is examined. At the same time, the density of the line is given in different vector types.

The sixth chapter contains the linear algebra part. Under this title, vector products, transformations, and Lorentz matrices are examined. The vector multiplication part is written clearly and simply to eliminate confusion in the literature. All possible situations are given with their results. The harmony between the inner product and vector product, which is not considered in the literature, is emphasized in this section. In which sources errors are found are explained with their reasons. In addition, information on special transformations and isometries is also given.

The seventh chapter is a comprehensive section examining curves in Lorentz space. In this section, curves in the space $\mathbb{R}^n_1$ are defined, and their properties are given. All special cases of curves are examined separately in the spaces $\mathbb{R}^4_1$, $\mathbb{R}^3_1$, and $\mathbb{R}^2_1$. Minkowski frenet formulas are given explicitly for timelike, spacelike, and null curves. All possibilities for null curves are evaluated with their results. In addition, clear titles are given for the involute evolute curve pair and the Bertrand curve pair. It has been explained that not every curve can be an involute evolute curve or a Bertrand curve pair. These errors are explained and special curve definitions are given with the original additions in this part of the book.

In the eighth chapter, information about surfaces is given in Minkowski space. The surface nomenclature is given carefully. All questions, such as which conditions are required on which surface and how vectors can be obtained, are answered in detail. Many topics, such as shape operator, gaussian transformation, and basic forms, are examined individually for each surface. The differences between them are noted.

In the ninth chapter, the concept of the manifold is defined. Manifold information on sub-manifold and hypersurface are examined in sub-headings. The tenth chapter contains the title of kinematics in Minkowski space. One parameter motion, Euler-Savary formulas, pole curves are expressed under the headings spacelike and timelike. Lorentz matrix multiplication, given in the previous sections, is combined with kinematics in this section. Therefore, spherical kinematic geometry is explained with the help of Lorentz matrix multiplication with its results in the space $\mathbb{R}^3_1$.

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